

DEUTERON ELECTRIC QUADRUPOLE AND OCTUPOLE POLARIZABILITIES

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The direct transition-matrix approach to determination of the electric polarizabilities of quantum bound systems developed in my recent work¹ is applied to study the electric multipole polarizabilities of a two-particle bound complex with a central interaction between the particles. Expressions for the electric quadrupole and octupole polarizabilities of the deuteron are derived and their values in the case of the S -wave separable interaction potential are calculated.

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1. Introduction

The electric polarizabilities of hadronic bound complexes are important physical quantities that characterize the deformation properties of composite structures containing information on the fundamental nuclear force between their constituents.

The result of the direct determination of the electric dipole polarizability of the deuteron by measuring deviations from the Rutherford scattering for d-²⁰⁸Pb elastic scattering below the Coulomb barrier,

$$\alpha_E(^2\text{H}) = 0.70 \pm 0.05 \text{ fm}^3 \text{ (Ref.2) ,}$$

is near to the result deduced from data for the total ²H photoabsorption cross section using the σ_{-2} sum rule,

$$\alpha_E(^2\text{H}) = 0.61 \pm 0.04 \text{ fm}^3 \text{ (Ref. 3) .}$$

The calculated values of the deuteron electric dipole polarizability, performed both with the simple separable potential that includes the tensor interaction between nucleons⁴ ($\alpha_E(^2\text{H}) = 0.6311 \text{ fm}^3$) and with more realistic nucleon-nucleon potentials⁵ ($\alpha_E(^2\text{H}) = 0.6328 \text{ fm}^3$) are within the experimental error of the result of Ref. 3, both the data^{2,3} being nearby.

The anisotropic character of the deuteron electric dipole polarizability caused by the presence of the tensor interaction between the neutron and proton has been explored both by the method of the traditional nuclear physics^{4,6} and in the framework of the chiral effective field theory^{7,8}. As it was shown in Ref. 4, the longitudinal component (with the electric field along the deuteron spin) α_{E1}^1 and the transverse component α_{E1}^0 are essentially differed between themselves: $\alpha_{E1}^1 = 0.669 \text{ fm}^3$ and $\alpha_{E1}^0 = 0.555 \text{ fm}^3$. (The abovementioned electric dipole polarizability $\alpha_{E1}(^2\text{H})$ is an averaged value of the longitudinal and transverse components: $\alpha_{E1} = \frac{2}{3}\alpha_{E1}^1 + \frac{1}{3}\alpha_{E1}^0$.)

Unfortunately, up to now the deuteron electric polarizabilities of the higher multipole order have not been experimentally determined yet.

The influence of the deuteron polarization (distortion) in the processes of scattering and disintegration of the deuteron in the Coulomb field of a nucleus at low energies was earlier studied in the papers by Abelishvili and Sitenko⁹ and by Clement¹⁰.

The aim of this work is to study theoretically the deuteron electric multipole polarizabilities by applying the direct transition-matrix approach formulated in the recent paper¹. In Section 2 the formula for the electric multipole polarizability of the two-body bound system composed of one charged and one neutral particles with a central interaction is derived. The use of the t -matrix approach in the case of the S -wave separable interaction potential is carried out in Section 3. Section 4 is devoted to the results of calculation of the deuteron quadrupole and octupole polarizabilities and discussion of them.

2. Transition-matrix approach to determination of the multipole polarizabilities of a two-body bound complex

The electric multipole polarizabilities of a bound complex $\alpha_{E\lambda}$ are the coefficients of the asymptotic expansion of the polarization potential of interaction between a charged particle 0 and the center of mass of the complex at distances ρ much more than the complex size,

$$V_{pol}(\rho_0) = -\frac{e_0^2}{2} \sum_{\lambda=1}^{\infty} \frac{\alpha_{E\lambda}}{\rho_0^{2\lambda+2}}, \quad (1)$$

where e_0 is the charge of the particle 0. In the case of a two-body complex that consists of a charged particle 1 and a neutral particle 2 (for instance, the deuteron), the electric polarizability of the multipole order 2^λ is given by

$$\alpha_{E\lambda} = -2 \langle \psi_0 | M_\lambda g^Q(-b) M_\lambda | \psi_0 \rangle. \quad (2)$$

Here ψ_0 is the wave function of the bound complex in the state with the energy $-b$, $g^Q(-b)$ is the "truncated" Green's operator at the energy $-b$,

$$g^Q(\varepsilon) = Qg(\varepsilon), \quad g(\varepsilon) = (\varepsilon - h^\circ - v_{12})^{-1}, \quad Q = 1 - P, \quad P = \psi_0 \langle \psi_0 |, \quad (3)$$

$g(\varepsilon)$ is the total Green's operator, P is the operator projecting on the ground state of the complex, h° and v_{12} are the kinetic energy and interaction potential operators of the relative motion of the constituent particles 1 and 2 of the complex, M_λ is the multipole operator

$$M_\lambda = e_1 r_1^\lambda P_\lambda(\hat{\mathbf{r}}_1 \cdot \hat{\boldsymbol{\rho}}) = e_1 \left(\frac{m_2}{m_{12}} \right) r^\lambda P_\lambda(\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\rho}}), \quad (4)$$

where e_1 is the charge of the particle 1 of the complex, \mathbf{r}_i is the radius-vector of the particle i of the complex relative to its center of mass, $\mathbf{r}_1 = \frac{m_2}{m_{12}} \mathbf{r}$, \mathbf{r} is the relative radius-vector between the particles of the complex, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $\boldsymbol{\rho}$ is the relative radius-vector between the particle 0 and the center of mass of the complex, m_i is the mass of the particle i , $m_{12} = m_1 + m_2$, $P_\lambda(x)$ is the Legendre polynomial. The unit vectors in (4) are marked by the hat.

In the momentum space (with $\mathbf{r} = i\hbar\nabla_{\mathbf{k}}$) the function $\langle \mathbf{k} | M_\lambda | \psi_0 \rangle$ that is contained in Eq. (2) can be written as

$$\langle \mathbf{k} | M_\lambda | \psi_0 \rangle = i^\lambda e_1 \left(-\frac{m_1}{m_{12}} \right)^\lambda \varphi_\lambda(k) P_\lambda(\hat{\mathbf{k}} \cdot \hat{\boldsymbol{\rho}}), \quad (5)$$

where

$$\varphi_\lambda(k) = (-1)^\lambda k^\lambda \left[\left(\frac{1}{k} \frac{d}{dk} \right)^\lambda \psi_0(k) \right]. \quad (6)$$

By its definition (3), the "truncated" Green's operator $g^Q(\varepsilon)$ has no singularity at $\varepsilon = -b$. To show explicitly cancellations of singularities in Eq. (3), it is worthwhile to express the total Green's function in terms of the transition operator,

$$g(\varepsilon) = g^\circ(\varepsilon) + g^\circ(\varepsilon)t(\varepsilon)g^\circ(\varepsilon). \quad (7)$$

Here $g^\circ(\varepsilon) = (\varepsilon - h^\circ)^{-1}$ is the free Green's operator and $t(\varepsilon)$ is the transition operator satisfying the Lippmann-Schwinger equation

$$t(\varepsilon) = v + v g^\circ(\varepsilon) t(\varepsilon). \quad (8)$$

In the case of a central interaction potential v , the operator $t(\varepsilon)$ can be decomposed into partial components $t_l(\varepsilon)$, each of them being characterized by a definite value of the orbital angular momentum of relative motion of particles l . The partial wave expansion of the transition matrix is given by

$$\langle \mathbf{k} | t^{(h)}(\varepsilon) | \mathbf{k}' \rangle = \sum_{l=1}^{\infty} (2l+1) t_l(k, k'; \varepsilon) P_l(\hat{\mathbf{k}} \cdot \hat{\mathbf{k}}'). \quad (9)$$

The S -wave partial component of the transition operator, $t_0(\varepsilon)$, has pole singularity at the point of the energy of the bound complex $\varepsilon = -b$ and can be represented as the sum of the pole t_0^P and smooth \tilde{t}_0 parts,

$$t_0(\varepsilon) = t_0^P(\varepsilon) + \tilde{t}_0(\varepsilon), \quad (10)$$

$$t_0^P(\varepsilon) = \frac{|u_0\rangle\langle u_0|}{\varepsilon + b} \quad (11)$$

with the vertex function

$$|u_0\rangle = -[g^\circ(-b)]^{-1} |\psi_0\rangle. \quad (12)$$

Cancelling the pole singularities of the operators $g(\varepsilon)$ and $Pg(\varepsilon)$ at $\varepsilon = -b$ in Eq. (3) we write the operator $g^Q(\varepsilon)$ as

$$\begin{aligned} g^Q(-b) = g^\circ(-b) & - g^\circ(-b) |\psi_0\rangle\langle\psi_0| - |\psi_0\rangle\langle\psi_0| g^\circ(-b) \\ & + g^\circ(-b) \tilde{t}(-b) g^\circ(-b). \end{aligned} \quad (13)$$

Here the operator $\tilde{t}(-b)$ denotes the smooth part of the transition operator t , it is the sum of the S -wave component $\tilde{t}_0(-b)$ and all the higher (with $l \geq 1$) partial

components of the transition operator $t(-b)$ that also have not singularities at this point,

$$\begin{aligned} \tilde{t}(-b) &= \tilde{t}_0(-b) + t^{(h)}(-b) , \\ \langle \mathbf{k} | t^{(h)}(-b) | \mathbf{k}' \rangle &= \sum_{l=1}^{\infty} (2l+1) t_l(k, k'; -b) P_l(\hat{\mathbf{k}} \cdot \hat{\mathbf{k}}') . \end{aligned} \quad (14)$$

One can readily see that the smooth S -wave operator $\tilde{t}_0(\epsilon)$ at the point $\epsilon = -b$ is separable,

$$\tilde{t}_0(-b) = | u_0 \rangle \left(-\frac{R_1}{b} \right) \langle u_0 | , \quad R_1 = b \langle u_0 | (b + h^\circ)^{-3} | u_0 \rangle , \quad (15)$$

with the formfactor in the form of the vertex function (12).

Substituting the expressions for the "truncated" Green's operator $g^Q(-b)$ (13) and for the smooth components of the transition operator $\tilde{t}(-b)$ and $\tilde{t}_0(-b)$, (14) and (15), into Eq. (2) and taking into account the relations

$$\begin{aligned} \langle \psi_0 | M_\lambda | \psi_0 \rangle &= e_2 \delta_{\lambda 0} , \\ \langle \psi_0 | g^\circ(-b) M_\lambda | \psi_0 \rangle &= \langle \psi_0 | M_\lambda g^\circ(-b) | \psi_0 \rangle^* = -e_2 \frac{R_1}{b} \delta_{\lambda 0} , \end{aligned} \quad (16)$$

we write the electric multipole polarizability of the two-body complex in the form

$$\begin{aligned} \alpha_{E\lambda} &= - 2 \langle \psi_0 | M_\lambda g^\circ(-b) M_\lambda | \psi_0 \rangle \\ &\quad - 2 \langle \psi_0 | M_\lambda g^\circ(-b) t^{(h)}(-b) g^\circ(-b) M_\lambda | \psi_0 \rangle , \quad \lambda = 1, 2, 3, \dots . \end{aligned} \quad (17)$$

After integrating in Eq. (17) over angular variables with the use of the formula (5) for the function $M_\lambda \psi_0$ and the partial expansion (14) for the operator $t^{(h)}(-b)$, we find the final expression for the electric multipole polarizability of the two-body complex with a central interaction between the constituent particles:

$$\begin{aligned} \alpha_{E\lambda} &= \frac{2}{2\lambda+1} e_1^2 \left(\frac{m_2}{m_{12}} \right)^{2\lambda} \left\{ \int_0^\infty \frac{dk k^2}{2\pi^2} \frac{|\varphi_\lambda(k)|^2}{\frac{k^2}{2\mu_{12}} + b} \right. \\ &\quad \left. - \int_0^\infty \int_0^\infty \frac{dk k^2 dk' k'^2}{4\pi^4} \frac{\varphi_\lambda^*(k) t_\lambda(k, k'; -b) \varphi_\lambda(k')}{\left(\frac{k^2}{2\mu_{23}} + b \right) \left(\frac{k'^2}{2\mu_{23}} + b \right)} \right\} . \end{aligned} \quad (18)$$

Here, the functions φ_λ , which are built out of derivatives of the wave function of the ground state of the complex according to Eq. (6), for the dipole ($\lambda = 1$), quadrupole ($\lambda = 2$) and octupole ($\lambda = 3$) polarizabilities take the form

$$\varphi_\lambda(k) = \begin{cases} -\psi'_0(k) & \text{for } \lambda = 1 , \\ +\psi''_0(k) - \frac{1}{k} \psi'_0(k) & \text{for } \lambda = 2 , \\ -\psi'''_0(k) + \frac{3}{k} \psi''_0(k) - \frac{3}{k^2} \psi'_0(k) & \text{for } \lambda = 3 , \end{cases} \quad (19)$$

and the corresponding partial components of the transition matrix satisfy the Lippmann-Schwinger equation

$$t_\lambda(k, k'; -b) = v_\lambda(k, k') - \int_0^\infty \frac{dk'' k''^2}{2\pi^2} v_\lambda(k, k'') \frac{1}{\frac{k''^2}{2\mu_{12}} + b} t_\lambda(k'', k'; -b) , \quad (20)$$

where $v_\lambda(k, k')$ is the partial component of the interaction potential $\langle \mathbf{k}|v|\mathbf{k}' \rangle$, μ_{12} is the reduced mass of the constituents.

The formula (18) is rigorous, it provides a possibility to calculate the multipole polarizabilities of both Coulomb and nuclear bound complexes with a central interaction between constituents. In accordance with Eqs. (18) and (19), to determine the 2^λ -pole polarizability of a two-particle complex, it is necessary to know only lower-order derivatives of the momentum-space bound-state wave function and a single partial component of the transition matrix at the negative energy of the bound state. (The P -, D -, and F -wave partial components of the t -matrix are needed to determine the dipole, quadrupole, and octupole polarizabilities, respectively.)

Compared to the approach, which is traditionally applied to calculate the electric dipole polarizability of atomic and nuclear two-body systems using the spectral expansion of the Green's operator $g(-b)$ in the complete system of eigenfunctions of the Hamiltonian $h = h^\circ + v_{12}$,

$$\alpha_{E1} = 2 \sum_{n \neq 0} \frac{|\langle \psi_n | \mathbf{D}_1 \cdot \hat{\boldsymbol{\rho}} | \psi_0 \rangle|^2}{\epsilon_n + b}, \quad (21)$$

where the summation is over all the possible discrete and continuous excited states n with the energy ϵ_n and the wave function ψ_n , t -matrix approach (18) is much simpler and mathematically clear since it rests only on the bound-state wave function and the corresponding partial component of the transition matrix at the negative energy of the state, both of them being real functions of relative momentum variables. With the use of the t -matrix approach, there is no need for calculation of more complicated complex continuum wave functions at all.

3. S -wave separable model

The expression (18) for the electric multipole polarizability of the two-body bound complex is obtained assuming that the interaction between constituents is central.

A further simplification of the formula (18) for the polarizability of the complex can be obtained describing the interaction between the charged particle 1 and the neutral particle 2 of the complex with the use of the pure S -wave separable potential¹¹

$$v_0(k, k') = -\nu u_0(k) u_0(k'). \quad (22)$$

In this case the wave function of the two-particle complex equals to

$$\psi_0(k) = \frac{u_0(k)}{\frac{k^2}{2\mu_{12}} + b}. \quad (23)$$

Using the Yukawa formfactor (Yu) in Eq. (22)

$$u_0(k) = \frac{(2\pi)^{3/2} \mathcal{N}}{k^2 + \beta^2} \quad (24)$$

and the relationship between the parameters ν and β that takes place if the potential (22) forms one bound state of the complex with the binding energy $b = \frac{\kappa^2}{2\mu_{12}}$,

$$\nu = \frac{\beta(\beta + \kappa)}{2\pi^2 \mu_{12}}, \quad (25)$$

we write the normalized wave function of the bound complex as

$$\psi_0(k) = \frac{C_0}{(k^2 + \kappa^2)(k^2 + \beta^2)}, \quad C_0 = 2\mu_{12}(2\pi)^{3/2}\mathcal{N} = \sqrt{8\pi\kappa\beta(\beta + \kappa)^3}. \quad (26)$$

In this case, the functions $\varphi_\lambda(k)$ (19) formed from derivatives of the wave function (26) are given by

$$\begin{aligned} \varphi_1(k) &= 2C_0 \frac{k(2k^2 + \beta^2 + \kappa^2)}{(k^2 + \beta^2)^2(k^2 + \kappa^2)^2}, \\ \varphi_2(k) &= 8C_0 \frac{k^2\{3k^4 + 3(\beta^2 + \kappa^2)k^2 + (\beta^4 + \beta^2\kappa^2 + \kappa^4)\}}{(k^2 + \beta^2)^3(k^2 + \kappa^2)^3}, \\ \varphi_3(k) &= 48C_0 \frac{k^3\{4k^6 + 6(\beta^2 + \kappa^2)k^4 + 4(\beta^4 + \beta^2\kappa^2 + \kappa^4)k^2 + (\beta^2 + \kappa^2)(\beta^4 + \kappa^4)\}}{(k^2 + \beta^2)^4(k^2 + \kappa^2)^4}. \end{aligned} \quad (27)$$

Note that the considered model (22) does not take account of the interactions in the higher orbital states. Also, the sole non-zero S -wave partial component of the transition matrix t_0 is not contained in the expression (18) for the multipole polarizabilities. Therefore, in this model the second term of the formula (18), in which t_λ is present only with $\lambda > 0$, is equal to zero, and the expression (18) is reduced to

$$\alpha_{E\lambda} = \frac{4}{2\lambda + 1} \left(\frac{m_2}{m_{12}} \right)^{2\lambda+1} \frac{m_1 e_1^2}{\hbar^2} \int_0^\infty \frac{dk k^2}{2\pi^2} \frac{|\varphi_\lambda(k)|^2}{k^2 + \kappa^2}. \quad (28)$$

Substituting then the functions (27) into the formula (28) and performing integration we obtain the following expressions for the dipole, quadrupole and octupole polarizabilities of the two-particle complex:

$$\alpha_{E1} = \frac{1}{12} \left(\frac{m_2}{m_{12}} \right)^3 \frac{m_1 e_1^2}{\hbar^2} \frac{3\beta^5 + 18\beta^4\kappa + 51\beta^3\kappa^2 + 96\beta^2\kappa^3 + 48\beta\kappa^4 + 8\kappa^5}{\beta^2(\beta + \kappa)^3\kappa^4}, \quad (29)$$

$$\begin{aligned} \alpha_{E2} &= \frac{1}{10} \left(\frac{m_2}{m_{12}} \right)^5 \frac{m_1 e_1^2}{\hbar^2} \frac{1}{\beta^4(\beta + \kappa)^5\kappa^6} \{5\beta^9 + 40\beta^8\kappa + 150\beta^7\kappa^2 + 360\beta^6\kappa^3 \\ &\quad + 645\beta^5\kappa^4 + 960\beta^4\kappa^5 + 640\beta^3\kappa^6 + 332\beta^2\kappa^7 + 96\beta\kappa^8 + 12\kappa^9\}, \end{aligned} \quad (30)$$

$$\begin{aligned} \alpha_{E3} &= \frac{9}{112} \left(\frac{m_2}{m_{12}} \right)^7 \frac{m_1 e_1^2}{\hbar^2} \frac{1}{\beta^6(\beta + \kappa)^7\kappa^8} \{35\beta^{13} + 350\beta^{12}\kappa + 1645\beta^{11}\kappa^2 \\ &\quad + 4900\beta^{10}\kappa^3 + 10605\beta^9\kappa^4 + 18270\beta^8\kappa^5 + 26915\beta^7\kappa^6 + 35840\beta^6\kappa^7 \\ &\quad + 26880\beta^5\kappa^8 + 18000\beta^4\kappa^9 + 9760\beta^3\kappa^{10} + 3616\beta^2\kappa^{11} + 800\beta\kappa^{12} + 80\kappa^{13}\}. \end{aligned} \quad (31)$$

In the case of the zero-range interaction ($\beta \rightarrow \infty$) the expressions (29) - (31) simplify to the form

$$\begin{aligned}
\alpha_{E1} &= \frac{1}{4} \left(\frac{m_2}{m_{12}} \right)^3 \frac{m_1 e_1^2}{\hbar^2} \frac{1}{\kappa^4} , \\
\alpha_{E2} &= \frac{1}{2} \left(\frac{m_2}{m_{12}} \right)^5 \frac{m_1 e_1^2}{\hbar^2} \frac{1}{\kappa^6} , \\
\alpha_{E3} &= \frac{45}{16} \left(\frac{m_2}{m_{12}} \right)^7 \frac{m_1 e_1^2}{\hbar^2} \frac{1}{\kappa^8} .
\end{aligned} \tag{32}$$

Note that the expression for the electric dipole polarizability of the two-body bound complex with the separable interaction (29) is in accordance with the corresponding formulae obtained by employing the Dalgarno-Lewis perturbation technique¹² in the paper by Friar and Fallieros¹³ and in the framework of the three-body formalism of the effective interaction between a charged particle and a complex in our papers^{14–16}. The t-matrix approach with S -wave separable interaction has been applied to determine the electric dipole polarizabilities of the triton and lambda hypertriton as two-cluster systems in the previous our works^{17,18}.

4. Numerical results and discussion

In this section we apply the formalism developed above to the deuteron as a bound complex composed of the proton (the particle 1, $e_1 = e_p$, $m_1 = m_p$, $\hbar^2/m_p e_p^2 = 28.819893$ fm is the proton Bohr radius) and the neutron (the particle 2, $m_2 = m_n$) with the binding energy $b = \kappa^2/2\mu_{pn}$. We calculate the dipole, quadrupole and octupole polarizabilities of the deuteron both in the model of the S -wave separable potential (22) with the Yukawa formfactor (24) and in the zero-range limit ($\beta \rightarrow \infty$).

We find the parameters of the separable potential (22) and (24), ν and β , by fitting them to the low-energy $p - n$ interaction data: the deuteron binding energy $b(^2\text{H})$,

$$b(^2\text{H}) = 2.224575(9) \text{ MeV} \quad (\text{Ref. 17}) \tag{33}$$

and the triplet $p - n$ scattering length $^3a_{pn}$ or the asymptotic S -wave normalization of the deuteron wave function $A_S(^2\text{H})$,

$$^3a_{pn} = 5.424(3) \text{ fm} \quad (\text{Ref. 18}) , \quad A_S(^2\text{H}) = 0.8845(8) \text{ fm}^{-1/2} \quad (\text{Ref. 19}) . \tag{34}$$

In the case of the model (22) and (24) we have

$$^3a_{pn} = \frac{2(\beta + \kappa)^2}{\beta\kappa(2\beta + \kappa)} , \quad A_S(^2\text{H}) = \frac{\sqrt{2\beta\kappa(\beta + \kappa)}}{\beta - \kappa} .$$

The results of the calculation are shown in Table 1. The existing experimental data for the deuteron dipole polarizability are mentioned in Section 1.

Table 1. The electric dipole α_{E1} , quadrupole α_{E2} and octupole α_{E3} polarizabilities of the deuteron, $\alpha_{E\lambda}(\text{H})$, calculated with the use of the separable interaction potential (22) (Yu) and the zero-range interaction model (ZR) with the parameters fitting to the $p-n$ data (33) - (34)

$p-n$ interaction [parameters]	ZR [$b(^2\text{H})$]	Yu [$b(^2\text{H}), {}^3a_{pn}$]	Yu [$b(^2\text{H}), A_S(^2\text{H})$]
$\alpha_{E1}(^2\text{H}), \text{fm}^3$	0.3776	0.6259	0.6292
$\alpha_{E2}(^2\text{H}), \text{fm}^5$	3.5247	5.9129	5.9462
$\alpha_{E3}(^2\text{H}), \text{fm}^7$	92.529	155.38	156.26

It is known¹³ that for the determination of the electric dipole polarizability of the deuteron the adequate description of the asymptotic behaviour of the deuteron wave function is of prime importance, since the weakly bound nucleons in the nucleus are with a great probability located at distances large compared to the interaction range. For the parameters of the potential fitted to both the $p-n$ bound-state data ($b(^2\text{H})$ and $A_S(^2\text{H})$), we find in Table 1 the following values of the deuteron electric polarizabilities

$$\alpha_{E1}(^2\text{H}) = 0.6292 \text{ fm}^3, \quad \alpha_{E2}(^2\text{H}) = 5.9462 \text{ fm}^5, \quad \text{and} \quad \alpha_{E3}(^2\text{H}) = 156.26 \text{ fm}^7. \quad (35)$$

The polarizabilities (35) are a little larger (by 0.5 %) than the ones obtained with the parameters fitted to the data b and ${}^3a_{pn}$ (when the potential (22) reproduces a somewhat lesser value of the asymptotic normalization: $A_S = 0.8820 \text{ fm}^{-1/2}$).

The deuteron dipole polarizability $\alpha_{E1}(^2\text{H})$ is further changed if the tensor $p-n$ interaction is taken into account. Thus, for example, with the use of separable tensor interaction potential^{22,13} (that reproduces $A_S = 0.8843 \text{ fm}^{-1/2}$) the dipole polarizability $\alpha_{E1}(^2\text{H})$ increases to 0.6311 fm^3 (Ref. 4) approaching the values obtained with the realistic potentials $\alpha_{E1}(^2\text{H}) = 0.6328(17) \text{ fm}^3$ (Ref. 5). As for the inclusion of the interaction in the higher partial states, notice that for the dipole polarizability of the deuteron $\alpha_{E1}(^2\text{H})$ it is more important the consideration of the tensor $p-n$ interaction (in ${}^3S_1 + {}^3D_1$ eigenstate) than that of the interaction in the P -wave state¹³. The tensor $p-n$ interaction also causes the anisotropy effect of the deuteron electric polarization—the dependence of the polarizability $\alpha_{E\lambda}^M(^2\text{H})$ on the projection of the total angular momentum (the spin) of the deuteron M that describes the direction of the spin relative to the electric field vector. The anisotropy effect of the electric dipole polarizability has been previously studied in Ref. 4. The application of the t -matrix approach to the study of the anisotropy of the induced electric quadrupole moment of the deuteron having also its own quadrupole moment ($Q = 0.2859(3) \text{ fm}^2$ (Refs. 23 and 24)) still remains to be carried out.

The advantage of the t -matrix formalism are clearly evident by the example of its application to the determination of the electric dipole polarizability of a simple two-particle system with long-range Coulomb interaction—the hydrogen atom. In this case, provided that the ratio of the electron mass to the proton mass tends to zero, the exact analytical value of the polarizability is known^{25–27}:

$\alpha_{E1}(\text{H}) = \frac{9}{2}a_B^3$, $a_B = \hbar^2/m_e e^2$ is the electron Bohr radius. With this, the formula (18) may be directly applied to the hydrogen atom. Substituting into the expression (18) the explicit analytical expression for the partial (with $l = 1$) component of the Coulomb transition matrix at negative energy $-b$ obtained previously in Ref. 28 we find calculating separately the first (analytically) and second (numerically) terms that

$$\alpha_{E1}(\text{H}) = \frac{7}{3}a_B^3 + 0.321066940 \text{ \AA}^3 = 0.666831341 \text{ \AA}^3 = 4.5 a_B^3.$$

As distinct from the polarizability of the two-particle nuclear system, for which in Eq. (18) the first term containing the free propagator is dominant as compared to the second one, the polarizability of the Coulomb system (with the long-range interaction) is determined by the sum of the two very nearly equal terms—with the free propagator and with the P -wave partial component of the transition matrix at the negative energy.

The developed approach to the problem of the determination of the electric multipole polarizabilities, which is based on the two-particle transition matrix, is found to be rather effective as compared with the traditional method that uses the spectral expansion of the Green's operator of the system. The results by Castillejo et al²⁹ testify that with the use of the spectral expansion all the virtual excited bound states contribute 81.4% of the magnitude of the electric multipole polarizability of the hydrogen atom, the rest is accounted for by the states of the continuum. With the application of the t -matrix approach the necessity of considering a large number of virtual states no longer arises at all. In the case of the dipole polarizability ($\lambda = 1$), beside the term with the free virtual propagator, the formula (18) contains only one term with the P -wave partial transition t -matrix at the negative energy. Moreover, the transition matrix at negative energies is a real quantity, as opposed to more complicate complex functions of the continuum, which are used in the traditional approach.

Existing advantages of the t -matrix approach motivate necessity of its further development with the aim of applying it for two-particle systems with noncentral interactions (the tensor $n - p$ interaction) to study the anisotropic properties of the dynamic quadrupole polarization of the deuteron. Also, the extension of the transition-matrix formalism to investigate the polarizabilities of more intricate three- and four-body nuclear systems based on the Faddeev and Faddeev-Yakubovsky integral equations is under way.

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